Worked Solutions

Edexcel C4 Paper E

1. (*a*) when $y = 1, 4x^2 + 3 = 12$

$$x^2 = \frac{9}{4}$$

$$x = \pm\frac{3}{2}$$ **(2)**

(*b*) differentiating, $8x + 6y\frac{dy}{dx} = 0 \qquad \frac{dy}{dx} = -\frac{8x}{6y} = -\frac{4x}{3y}$

at $\left(\frac{3}{2}, 1\right)$ gradient $= -\dfrac{4 \times \frac{3}{2}}{3} = -2$

at $\left(-\frac{3}{2}, 1\right)$ gradient $= 2$ **(4)**

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2. $(8 + x)^{\frac{1}{3}} = \left[8\left(1 + \frac{x}{8}\right)\right]^{\frac{1}{3}} = 2\left(1 + \frac{x}{8}\right)^{\frac{1}{3}} = 2\left[1 + \frac{1}{3}\left(\frac{x}{8}\right) + \dfrac{\frac{1}{3}\left(-\frac{2}{3}\right)}{2}\left(\frac{x}{8}\right)^2 + \dots\right]$

$$= 2 + \frac{x}{12} - \frac{1}{288}x^2 + \dots$$ **(4)**

(*b*) for $\left(8 + 3m + m^2\right)^{\frac{1}{3}}$, let $3m + m^2 = x$

$$\left(8 + 3m + m^2\right)^{\frac{1}{3}} = 2 + \left(\frac{3m + m^2}{12}\right) - \frac{1}{288}\left(3m + m^2\right)^2$$

$$= 2 + \frac{1}{4}m + \frac{1}{12}m^2 - \frac{1}{288} \cdot 9m^2 + \dots$$

$$= 2 + \frac{1}{4}m + \frac{1}{12}m^2 - \frac{1}{32}m^2 = 2 + \frac{1}{4}m + \frac{5}{96}m^2.$$ **(3)**

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3. (*a*) $\dfrac{dy}{dx} = \dfrac{\frac{1}{2} \cdot 2\cos 2\theta}{-\sin\theta} = -\dfrac{\cos 2\theta}{\sin\theta}$

at $\theta = \frac{\pi}{6}$, gradient $= -\dfrac{\cos\frac{\pi}{3}}{\sin\frac{\pi}{6}} = -\dfrac{\frac{1}{2}}{\frac{1}{2}} = -1$

(*b*) at $\theta = \frac{\pi}{6}, x = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ and $y = \frac{1}{2}\sin\frac{\pi}{3} =$

equation of tangent is $y - \frac{\sqrt{3}}{4} = -1\left(x - \frac{\sqrt{3}}{2}\right)$

$$4y - \sqrt{3} = -4x + 2\sqrt{3}$$

$$4y + 4y = 3\sqrt{3}$$

(*c*) $y^2 = \frac{1}{4}\sin^2 2\theta = \frac{1}{4}(2\sin\theta\cos\theta)^2 = \frac{1}{4} \cdot 4\sin^2\theta$ co

$\therefore \quad y^2 = (1 - x^2)(x^2)$

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4. (*a*) $(0, 10)$

(*b*) $\dfrac{dy}{dx} = -10(-k)e^{-kx}$

at $x = 0$, gradient $= 10k$

$$10k = 5 \Rightarrow k = \frac{1}{2}$$

(*c*) area $= \displaystyle\int_0^4 \left(20 - 10\,e^{-\frac{1}{2}x}\right)\,dx = \left[20x + 20\,e^{-\frac{1}{2}x}\right]$

$$= 80 + 20\,e^{-2} - (0 + 20) = 60 + \frac{20}{e^2}$$

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5. (*a*) when $t = 0, \theta = 300 - 270e^{\circ}$

$$\theta = 30 \qquad \textbf{(1)}$$

(*b*) as $t \to \infty, \theta \to 300$ **(2)**

(*c*) $200 = 300 - 270\,e^{-0.05t}$

$$270\,e^{-0.05t} = 100$$

$$\ln e^{-0.05t} = \ln\left(\frac{100}{270}\right)$$

$$-0.05t = \ln\left(\frac{100}{270}\right)$$

$$t = 19.9 \text{ minutes} \qquad \textbf{(3)}$$

(*d*) $\frac{d\theta}{dt} = -270(-0.05)\,e^{-0.05t}$

when $t = 2$, $\frac{d\theta}{dt} = 270 \times 0.05 \times e^{-0.1}$

$$= 12.2^{\circ}\text{ C/min} \qquad \textbf{(3)}$$

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6. (*a*) $\frac{2}{1-x} - \frac{2}{2-x}$ ('cover up' rule) **(2)**

(*b*) we have $\frac{2}{1-x} - \frac{2}{2(1-\frac{x}{2})} = 2(1-x)^{-1} - \left(1 - \frac{x}{2}\right)^{-1}$

$$= 2\left[1 + (-1)(-x) + \frac{(-1)(-2)}{2}(-x)^2 + \ldots\right]$$

$$- \left[1 + (-1)\left(\frac{-x}{2}\right) + \frac{(-1)(-2)}{2}\left(\frac{-x}{2}\right)^2 + \ldots\right]$$

$$= 2 + 2x + 2x^2 - 1 - \frac{x}{2} - \frac{x^2}{4}$$

$$= 1 + \frac{3}{2}x + \frac{7}{4}x^2 \qquad \textbf{(6)}$$

(*c*) $\int_0^{\frac{1}{2}} \left(\frac{2}{1-x} - \frac{2}{2-x}\right) dx = \Big[-2\ln(1-x)$ …

$$= -2\ln\frac{1}{2} + 2\ln\frac{3}{2} - \ldots$$

$$= -2\ln 2^{-1} + 2\ln\frac{3}{2} - 0 - \ldots$$

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7. (*a*) let $\angle ABC = \theta, \overrightarrow{BA} = \begin{pmatrix} -4 \\ 0 \\ -3 \end{pmatrix}, |\overrightarrow{BA}| = \sqrt{4^2 + 3^2} = 5$ …

$$\overrightarrow{BC} = \begin{pmatrix} -6 \\ 1 \\ 3 \end{pmatrix}, |\overrightarrow{BC}| = \sqrt{6^2 + 1^2 + 3 \ldots}$$

$$\overrightarrow{BA} \cdot \overrightarrow{BC} = |\overrightarrow{BA}| \times |\overrightarrow{BC}| \cos\theta$$

$$\begin{pmatrix} -4 \\ 0 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} -6 \\ 1 \\ 3 \end{pmatrix} = 5\sqrt{46}\cos\theta \qquad 24 - 9 = 5\sqrt{46}\,c\ldots$$

(*b*) area of $\triangle ABC = \frac{1}{2}|\overrightarrow{BA}| \times |\overrightarrow{BC}| \sin\theta$

$$\text{area} = \frac{1}{2} \cdot 5 \times \sqrt{46} \cdot \frac{\sqrt{37}}{\sqrt{46}}$$

$$= \frac{5}{2}\sqrt{37} \qquad \sin\theta \ldots$$

(*c*) $\overrightarrow{AC} = \begin{pmatrix} -2 \\ 1 \\ 6 \end{pmatrix}$ $\quad \overrightarrow{OD} = \begin{pmatrix} 4 \\ -2 \\ -12 \end{pmatrix}$

$\overrightarrow{OD} = -2\begin{pmatrix} -2 \\ 1 \\ 6 \end{pmatrix}$, so AC is parallel to OD.

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8. (*a*) Let $I = \int_0^1 \frac{x}{(2x+1)^2}\,dx$

let $t = 2x + 1$

$\frac{dt}{dx} = 2 \Rightarrow dx = \frac{1}{2}\,dt$

$x = \frac{1}{2}(t-1)$

when $x = 1, \quad t = 3$
$x = 0, \quad t = 1$

$$\therefore \quad I = \frac{1}{2}\int_1^3 \frac{1}{2}\frac{(t-1)}{t^2}\,dt$$

$$= \frac{1}{4}\int_1^3 \left(\frac{1}{t} - t^{-2}\right) dt = \frac{1}{4}\left[\ln t + \frac{1}{t}\right]_1^3$$

$$= \frac{1}{4}\left[\ln 3 + \frac{1}{3} - (\ln 1 + 1)\right]$$

$$= \frac{1}{4}\left[\ln 3 - \frac{2}{3}\right]$$

(7)

(*b*) $$\int_1^e x^2 \ln x \;dx = \int_1^e \ln x \frac{d}{dx}\left(\frac{x^3}{3}\right) dx$$

$$= \left[\frac{x^3}{3}\ln x\right]_1^e - \int_1^e \frac{x^3}{3}\cdot\frac{1}{x}\;dx$$

$$= \left(\frac{e^3}{3}\ln e - 0\right) - \left[\frac{x^3}{9}\right]_1^e$$

$$= \frac{e^3}{3} - \left(\frac{e^3}{9} - \frac{1}{9}\right) = \frac{3e^3 - e^3 + 1}{9}$$

$$= \frac{2e^3 + 1}{9}$$